Estimating Complete Hysteresis Loops of Permanent Magnets Using Excel

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Abstract
Because of their high intrinsic coercivity values, measuring a complete major hysteresis curve of permanent magnets, especially, any rare earth magnet, is a difficult task. To overcome this deficiency, this paper describes a simple way to estimate the complete loop from a few measured parameters. Comparisons to real materials show this method has usefulness in estimating curves, or even for creating datasheets or graphics.

Key words: permanent magnet, magnetic hysteresis

I. INTRODUCTION

While permanent magnets are great enablers of modern technology, determining their magnetic properties has always been challenging. This is particularly true for rare earth permanent magnets, based on either SmCo or NdFeB. The typical intrinsic coercivity values of these latter materials often exceeds the maximum magnetic field which can be applied by an electromagnet. As a result, compromised magnetic testing methods are routinely employed,[1] and full major hysteresis loops are rarely encountered in actual practice for permanent magnets.

The literature is replete with many papers describing elegant methods to derive hysteresis curves of ferro- and ferrimagnetic materials from first principles.[2-9] The common theme among these models is that they describe magnetic hysteresis in terms of *intrinsic* properties i.e., M_s, H_A, T_c. The model described in this paper is completely different in that it is based on *extrinsic* properties, i.e., B_r, H_k, H_{cJ} and μ_r (the slope of the B vs. H curve in the second quadrant, the recoil permeability), all process-sensitive properties. Since the starting points are not the same, the model presented here is not meant to replace or supersede the earlier works. It is simply presented as a tool for estimating what a complete hysteresis loop might look like using a spreadsheet if adequate magnetic fields were available for the measurement.

II. ASSUMPTIONS

The model is based on several assumptions about the nature and behavior of a permanent magnet.

- The grains are non-interacting
- The grains are all the same size
- Each grain has a unique magnetic field value where its magnetization reverses, called H_{flip}
- The values of H_{flip} for the grains in a magnet obey a Gaussian or normal distribution
- The slope of the M vs. H curve near B_r is constant and related to the alignment of the grains in the magnet

With these assumptions, a few things can be stated mathematically about the grain reversal process and their connection to the Gaussian distribution. At B_r, none of the grains have flipped. The intrinsic coercivity, H_{cJ} is where exactly 50% of the grains have reversed their magnetization. Therefore, H_{cJ} is both the mean and median of the normal distribution. Since H_k is defined as the magnetic field required to reduce the magnetization by 10% from B_r, it means that 5% of the grains have reversed at this field. To apply a normal distribution, the standard deviation is also necessary. For the cumulative probability function to be 50% at H_{cJ} and 5% at H_k, the standard deviation of the distribution must meet the following condition

$$\sigma = \frac{H_{cJ} - H_k}{1.645} \qquad (1)$$

The effect of alignment is typically small for well-aligned anisotropic magnets but is more apparent for isotropic magnets, such as bonded NdFeB. Near B_r, the slope of the M vs. H curve is nearly zero for anisotropic materials and somewhat greater than zero for isotropic materials. We will model this slope as (μ_r-1) and will treat it as a constant.

It is certainly appropriate to ask if these assumptions are reasonable. The most questionable assumption is the first one, that the grains do not interact. Adding the complexity of interacting grains has not been explored for this work but it appears to make the model unwieldy for a simple spreadsheet. If the model can be improved, this is the place to look. Assuming the particles are all the same size is clearly untrue but not so important. The grain sizes likely also follow a normal distribution. Again, it would add more complexity to the model to include this fact, but it is questionable that it would change the results significantly or add much insight to them. The assumption that the reversal field, H_{flip}, obeys a Gaussian distribution seems realistic, although H_{flip} might not be precisely unique in each grain because of interactions with neighboring grains, or other factors. The final assumption is saying that the slope of the M vs H curve is a constant. This behavior appears to be true around B_r, but it is likely a weak function of H elsewhere, making this the second most questionable assumption.

III. Equation

Combining the two independent effects: the distribution of the reversals and the effect of alignment, the following equation is obtained describing the polarization, $J=\mu_0 M$ as a function of H

$$J(H)=(B_r)\,erf\left[\frac{1.645(H_{cJ}-H)}{\sqrt{2}(H_{cJ}-H_k)}\right]+\mu_0 H(\mu_r-1) \qquad (2)$$

Equation (2) applies to half of the hysteresis loop. The results are shown in Figure 1 for a typical set of parameters.

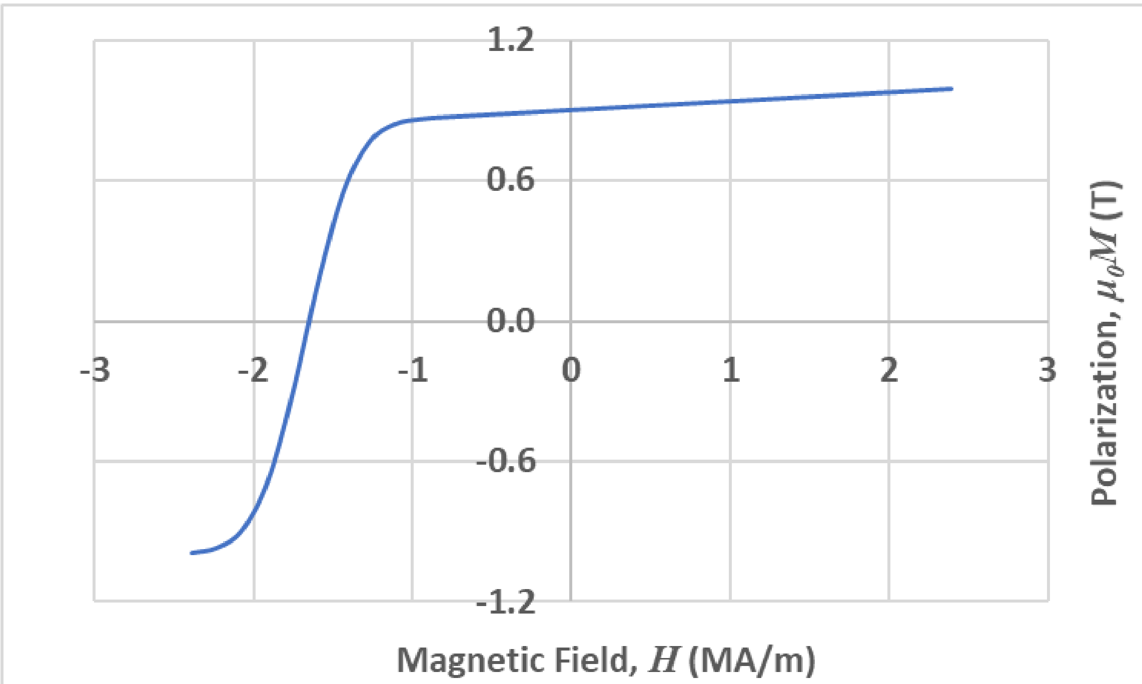

Figure 1. The results of equation (2) for B_r=0.9 T, H_k= 1.27 MA/m, H_{cJ}=1.67 MA/m and μ_r=1.03

The first term in Equation (2) captures the effect of the normal distribution. The error function is used here because we are interested in the cumulative probability that the grains have reversed to the left of B_r in Figure 1. It is the dominant term of the equation in the second quadrant. The second term captures the adjustment for alignment and is a smaller correction. Its effect is easier to see in the first quadrant, with a slight positive slope, where the probability term is constant, fixed at 1.

A similar equation is applicable for the other half of the hysteresis loop, when the variable H is replaced by -H

$$J(H)=-(B_r)\,erf\left[\frac{1.645(H_{cJ}-H)}{\sqrt{2}(H_{cJ}-H_k)}\right]-\mu_0 H(\mu_r-1) \qquad (3)$$

Equation (3) is plotted in Figure 2, using the same parameters as Figure 1.

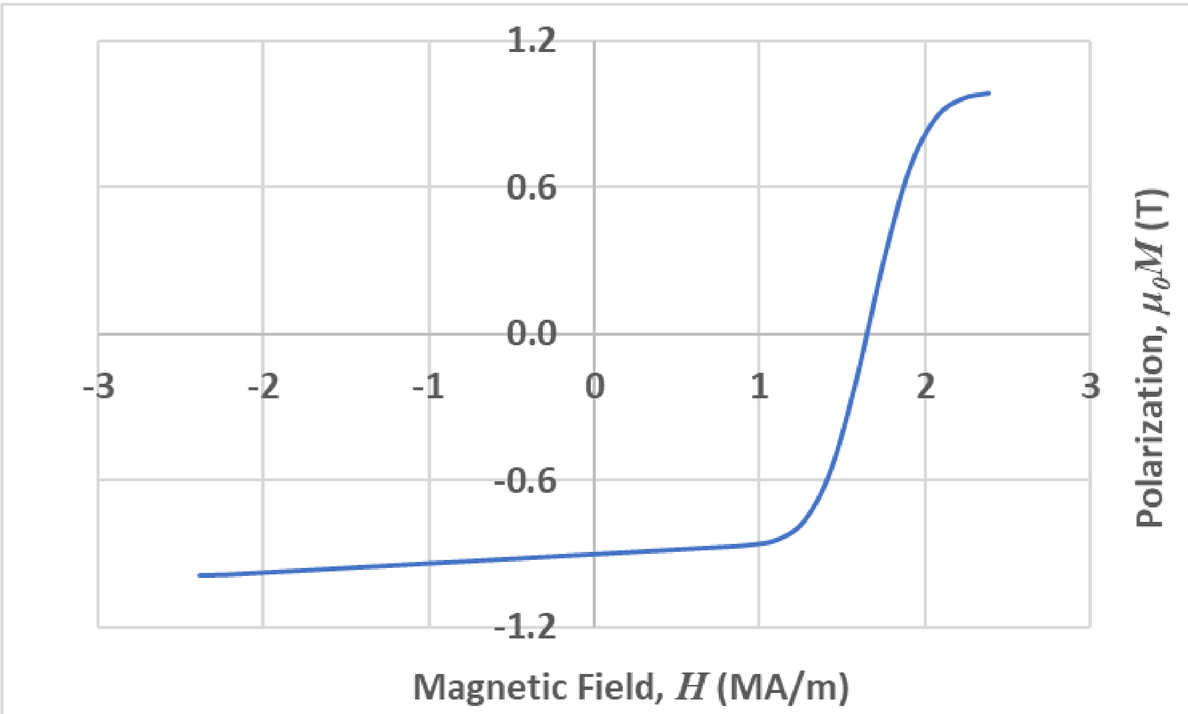

Figure 2. A plot of Equation (3) using the same magnetic data as Figure 1 to create the other half of the hysteresis loop.

Independent work by Martinez-Garcia et al.[10] on nanocrystalline ribbons using a similar methodology arrives at a very similar equation. See their equation 5.

IV. Comparisons to Real Materials

Using Equations (2) and (3), the following figures show calculated and measured curves for sintered NdFeB, bonded NdFeB, SmCo 2-17 and ferrite magnets. In each of the following figures, the blue curve is calculated, and the red curve is measured. Note that typical measured curves only report data for the second quadrant because of the limitations mentioned in the Introduction.

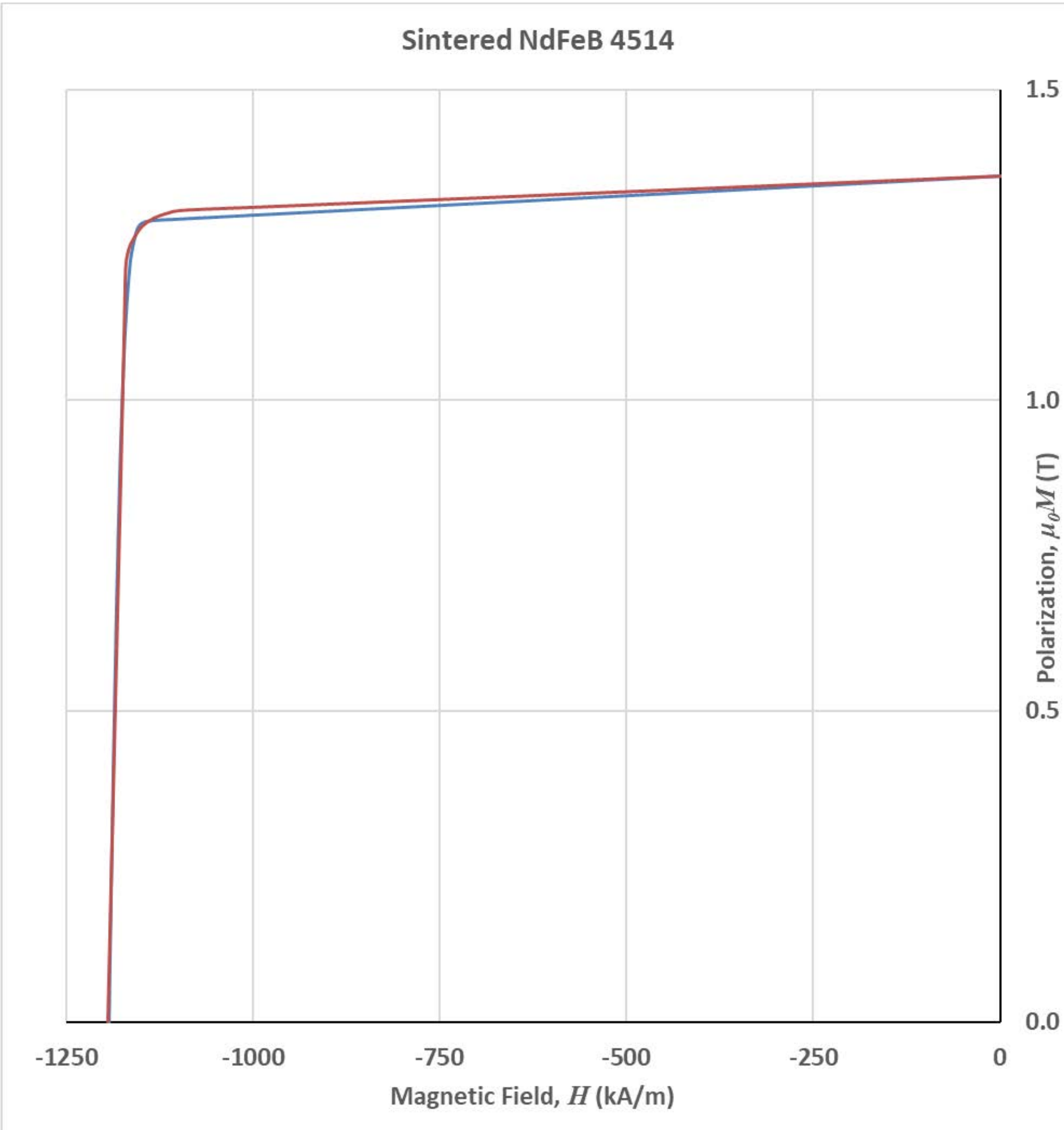

Figure 3. The second quadrant demagnetization curve for a sintered NdFeB sample. The magnetic data used to generate the calculated curve are B_r = 1.36 T, H_k = 1.17 MA/m, H_{cJ} = 1.19 MA/m and μ_r = 1.05. The correlation for these curves is 0.99.

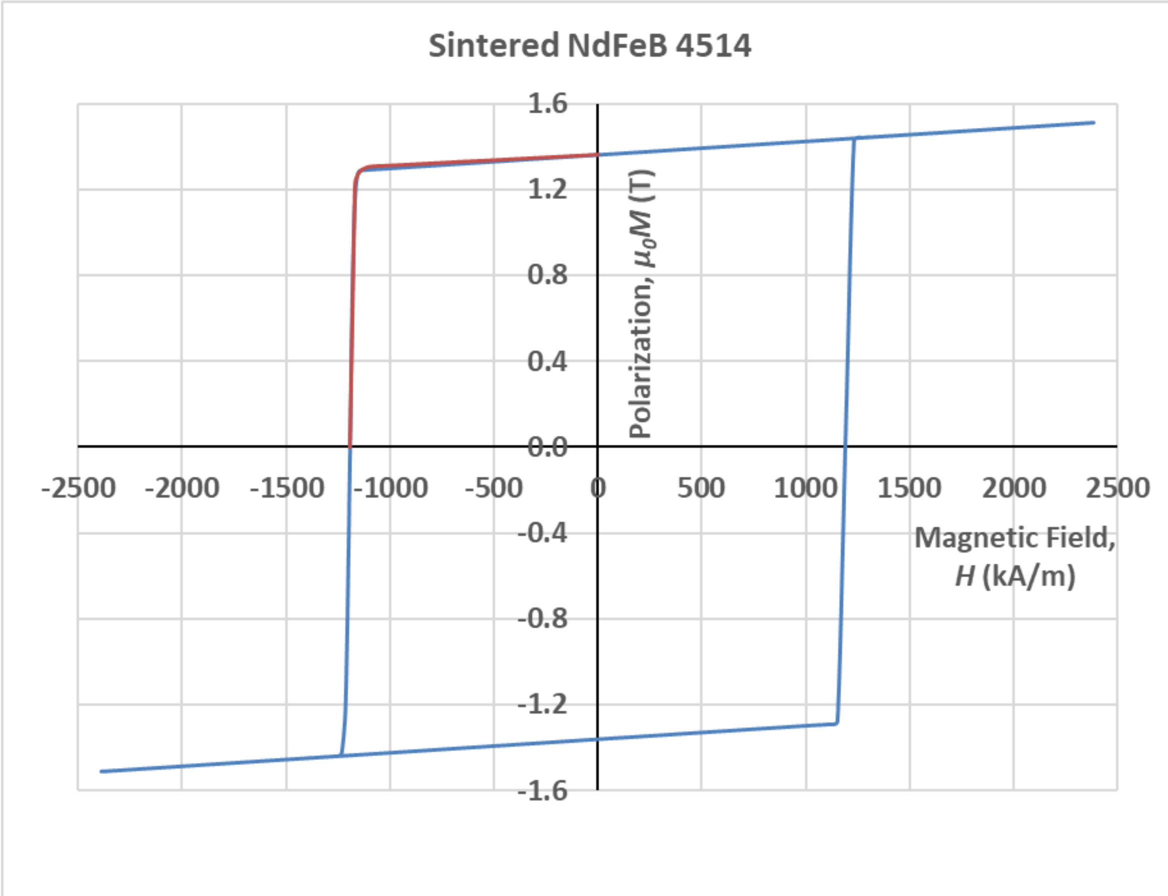

Figure 4. The full hysteresis loop for the same sample as Figure 3.

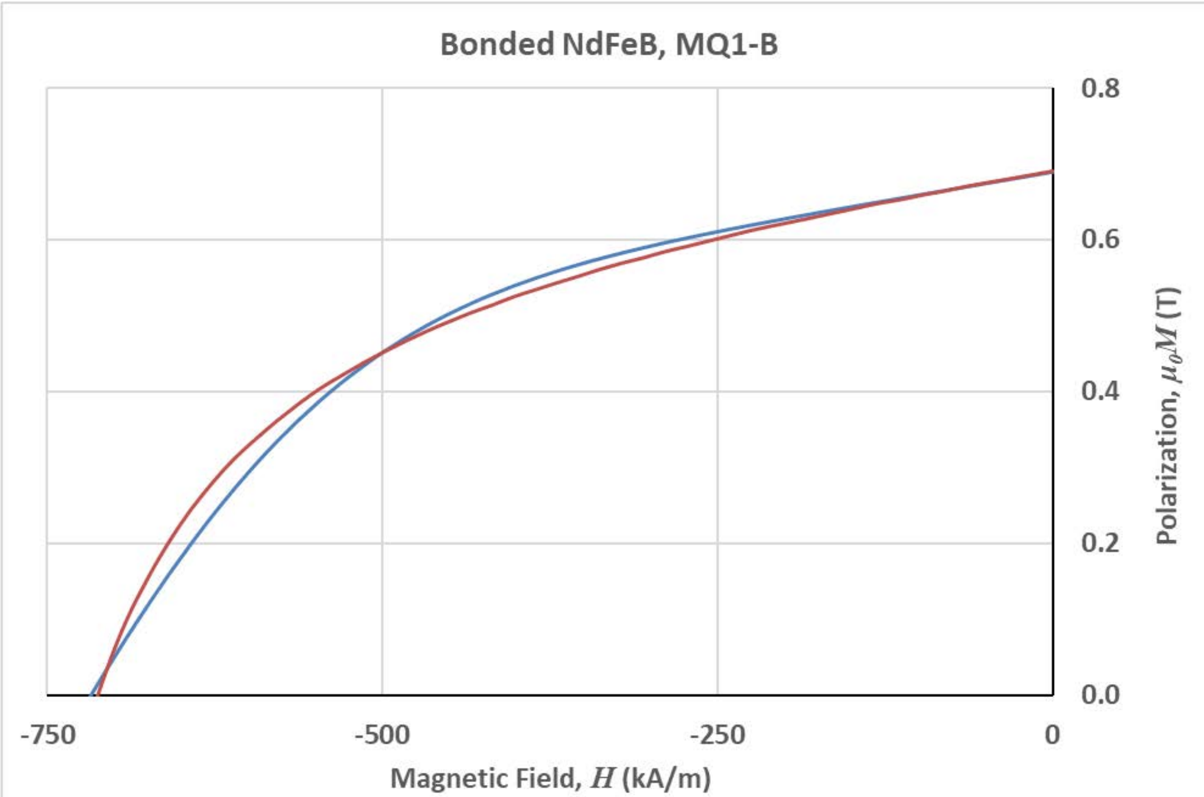

Figure 5. A second quadrant demagnetization curve for a bonded NdFeB sample. The magnetic data used to generate the calculated curve are B_r = 0.69 T, H_k = 0.48 MA/m, H_{cJ} = 0.80 MA/m and μ_r = 1.24. The correlation for these curves is 0.99.

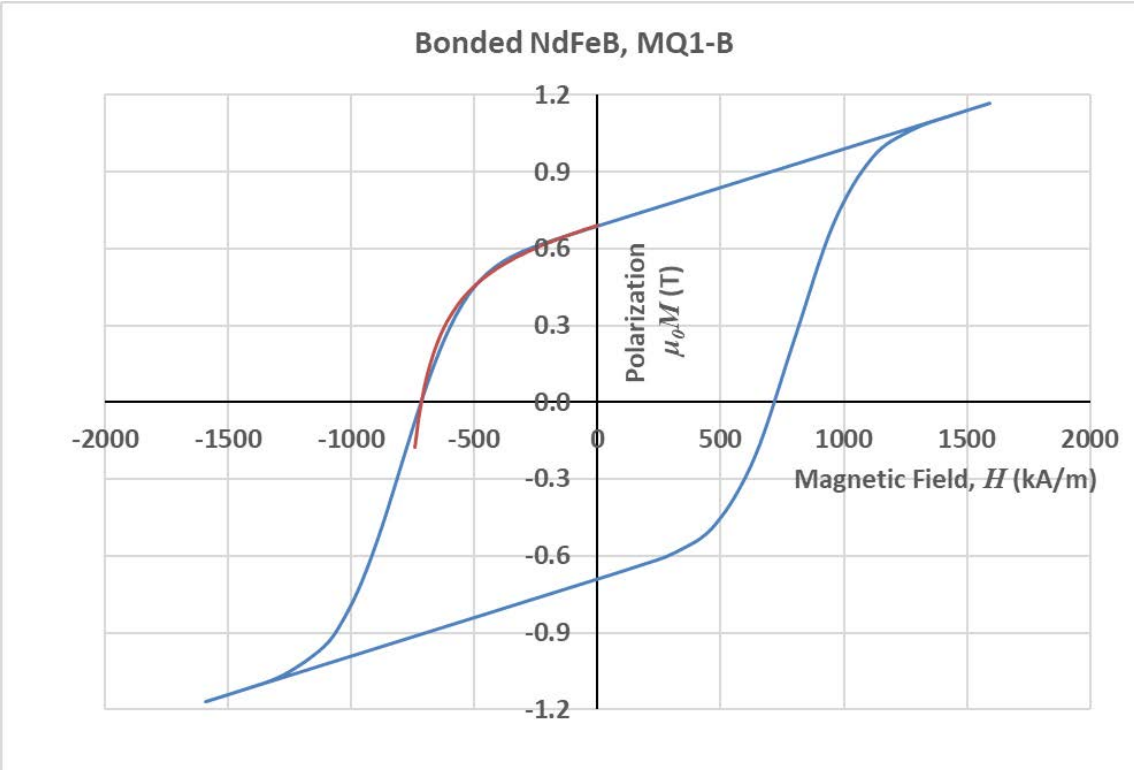

Figure 6. The full hysteresis loop for the same sample as Figure 5.

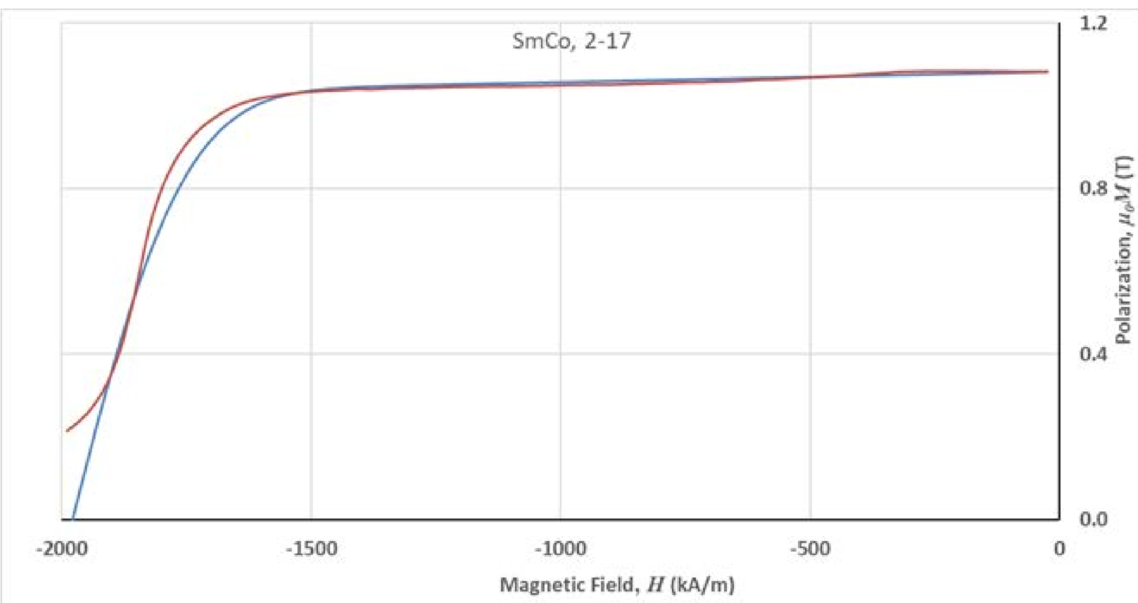

Figure 7. A second quadrant demagnetization curve for an SmCo 2-17 sample. The magnetic data used to generate the calculated curve are B_r = 1.08 T, H_k = 1.69 MA/m, H_{cJ} = 1.99 MA/m and μ_r = 1.02. The correlation for these curves is 0.99

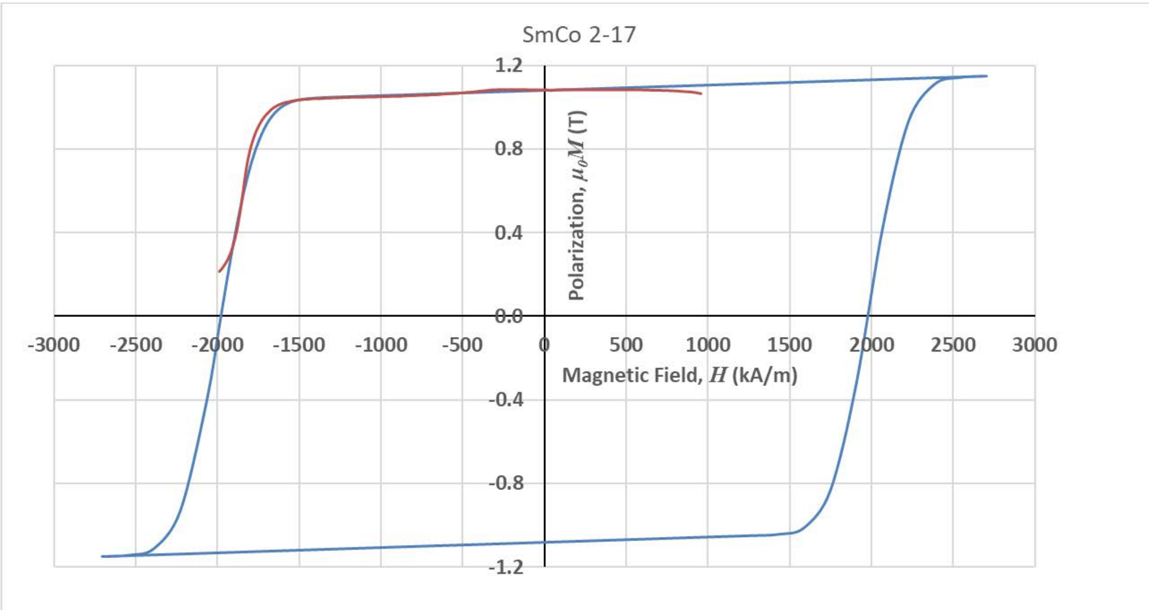

Figure 8. The full hysteresis loop for the same sample as Figure 7.

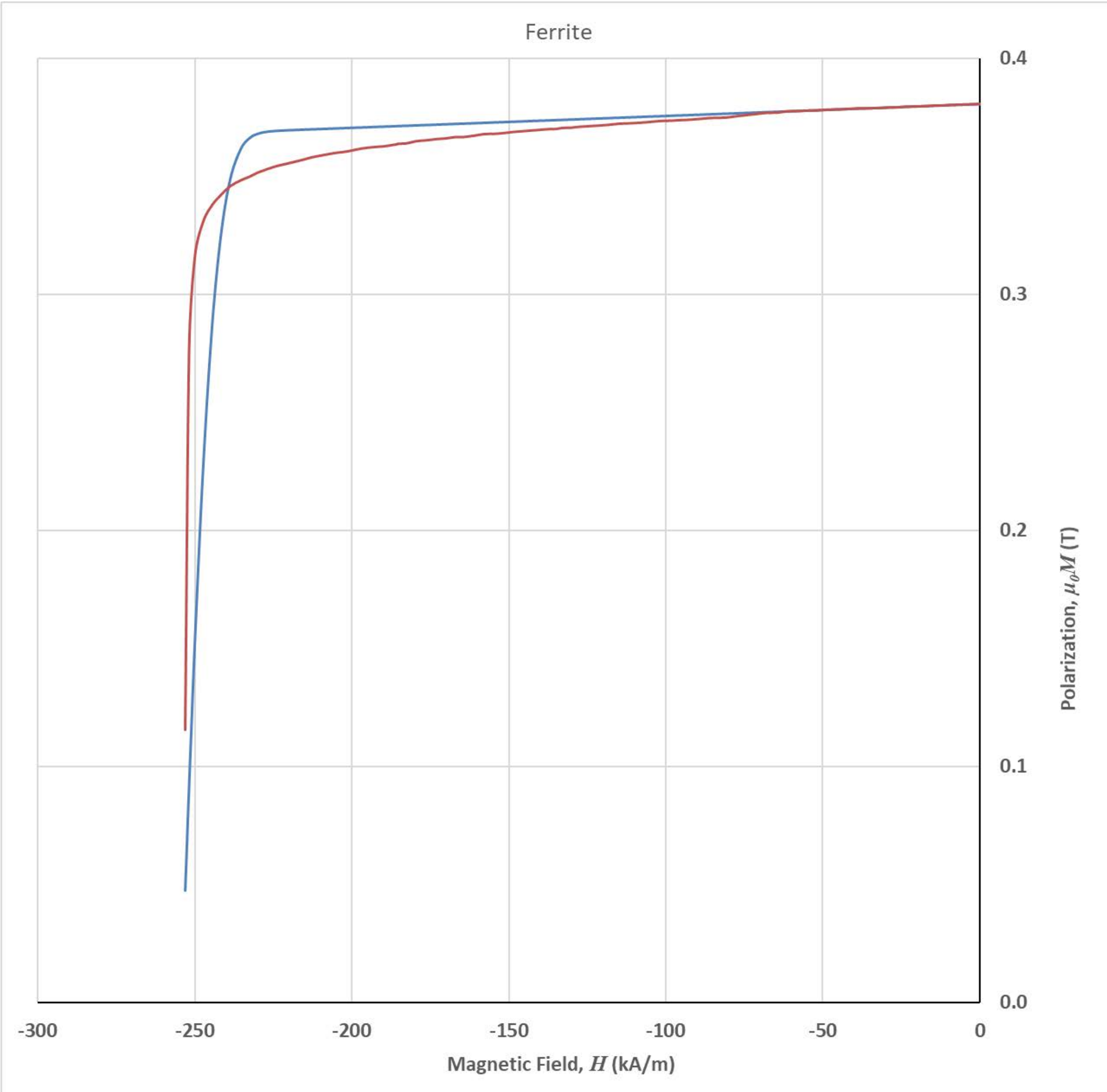

Figure 9. A second quadrant demagnetization curve for a ferrite sample. The magnetic data used to generate the calculated curve are B_r = 0.38 T, H_k = 240 kA/m, H_{cJ} = 255 kA/m and μ_r = 1.05. The correlation is 0.89.

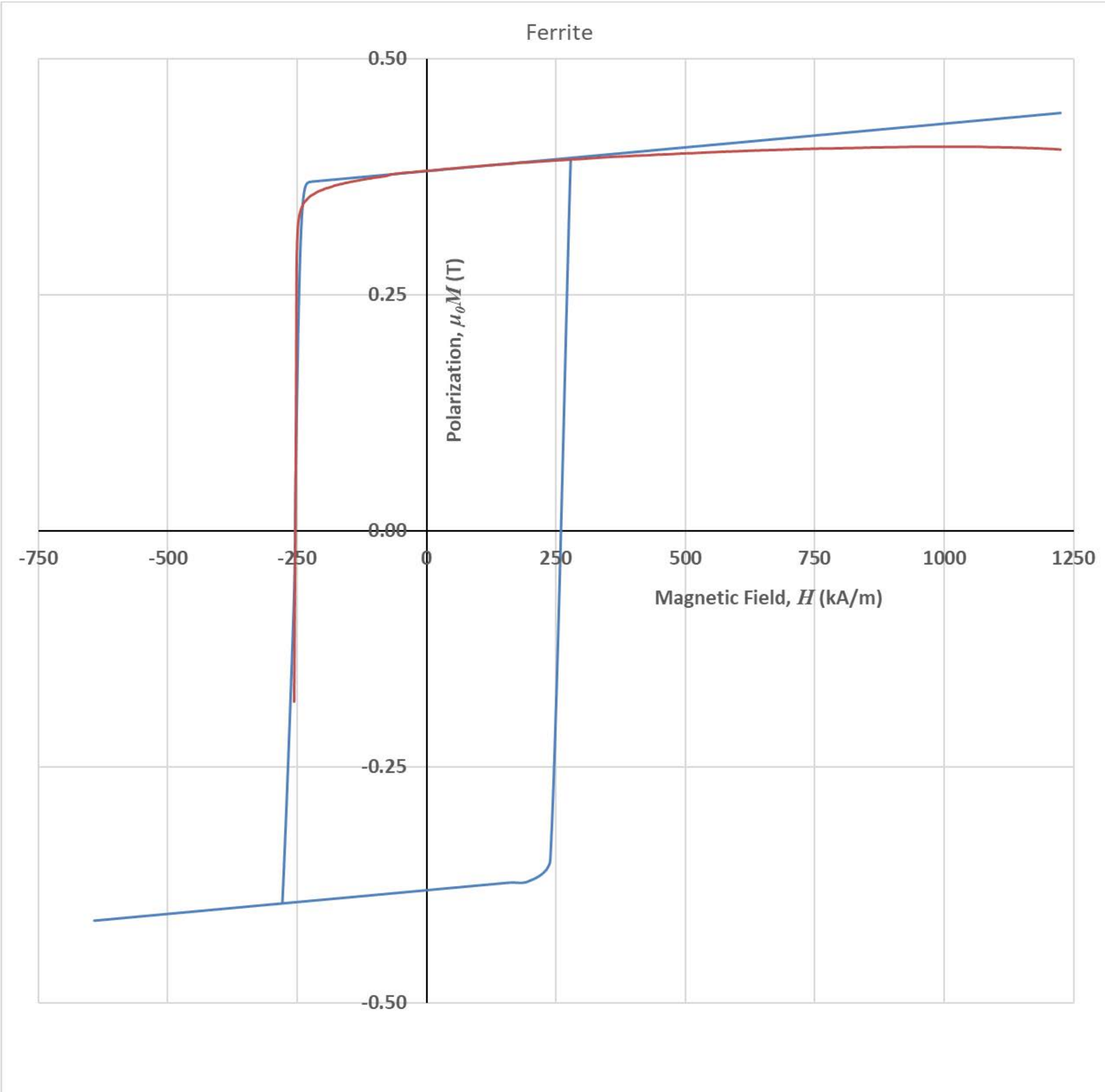

Figure 10. The full hysteresis loop for the sample shown in Figure 9.

V. CONCLUSIONS

The model shows very good fit for a few real materials. The correlation was 0.99 for all the rare earth samples and 0.89 for the ferrite sample. It is likely that the assumptions for the model are more realistic for rare earth magnets than hard ferrites.

While this model was constructed around using H_k and H_{cJ} as two of the key parameters, other points on the curve could be substituted for the same type of analysis should the applied magnetic field be limited in magnitude, as frequently happens. See figures 7 and 8 for an example of a material without a measured value of H_{cJ}. This approach might be helpful in trying to measure large quantities of magnets, in either a production or a magnet recycling environment. It also could be helpful for curve fitting of hysteresisgraph measurements.

The model is suitable for drawing full hysteresis loops for illustrations or creating a database of properties for an entire family of materials.

A template of the spreadsheet is available for people interested in trying the model. Please contact the author for a copy strout@ieee.org.

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