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A Second Look at the Reversible Temperature Coefficients of Permanent Magnets

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Abstract—In a recent publication, Lewis et al. [2014] introduced a new equation to define the reversible temperature coefficient of H_{cJ}, commonly called β. While the new equation is mathematically correct, in practice it leads to numerical values differing from the original definition, in some cases substantially. This paper investigates the old and new definitions, and demonstrates why the old equation is a better choice for reporting the reversible change in H_{cJ} with temperature.

I. INTRODUCTION

The introduction of Nd-Fe-B magnets over three decades ago marked the beginning of an era of permanent magnets with modest Curie temperatures (T_C). Consequently, being able to quantify, compare and predict the reversible changes in magnetic properties with temperature is an important practical consideration. Similarly, the thermal characteristics of any newly developed materials must be explored for benchmarking purposes.

Two coefficients commonly used for this purpose are defined in the equations below,

$$\alpha = \frac{1}{B_r}\left(\frac{\Delta B_r}{\Delta T}\right) \tag{1}$$

$$\beta = \frac{1}{H_{cJ}}\left(\frac{\Delta H_{cJ}}{\Delta T}\right) \tag{2}$$

where α is the reversible temperature coefficient of the remanence, B_r, and β is the reversible temperature coefficient of the intrinsic coercivity, H_{cJ}. In practice, the coefficients are calculated by selecting data at two different temperatures, commonly room temperature and an elevated temperature. As we shall see, neither B_r nor H_{cJ} vary linearly with temperature. Therefore, it is important to report the temperature range used for the measurement, otherwise the results can be misleading.

The coefficients α and β are frequently used to compare two competing materials with otherwise comparable other magnetic properties. Materials with values of α and β closer to zero are considered desirable. Consequently, maintaining rigorous, "apples to apples" or like to like comparisons is very important.

The following equation was used by Lewis et al. [2014] to define β of a particular Fe-Ni phase found in meteorites

$$\beta = \frac{d\left(\ln H_{cJ}\right)}{dT} \tag{3}$$

At first glance, this equation appears somewhat different than equation (2). However, this discrepancy is easily clarified with some rudimentary Calculus. If we wish to use equation (2) to determine the value of β at a specific temperature, we would write

$$\beta = \frac{1}{H_{cJ}}\left(\frac{dH_{cJ}}{dT}\right) \tag{4}$$

But equations (3) and (4) are equivalent, since basic differential Calculus tells us that

$$\frac{d\left(\ln u\right)}{dx} = \frac{1}{u}\frac{du}{dx} \tag{5}$$

(Kahn Academy [2017] offers a more complete explanation of this derivative.) The critical difference between equation (2) and equations (3) and (4) is the magnitude of ΔT. The ΔT in equation (2) can easily be on the order of 100 K, as it is typically used. In contrast, because equations (3) and (4) are derivatives, they must be considered as ΔT approaches zero.

An obvious question to consider is: does the choice of equation to determine β affect the result?

II. APPLICATION TO REAL MATERIALS

In the Tables below, the values for β are calculated for two grades of sintered Nd-Fe-B sold by Vacuumschmelze as VACODYM grades 3230 and 4514. The 3230 grade has a relatively large value of H_{cJ} and a relatively small value of β, while the 4514 grade has more modest values of both H_{cJ} and β. For comparison, both equations (2) and (3) are used to find β in the following tables. In this analysis, equation (3) needs to be rewritten in the Δ format, in light of the discrete nature of the experimental data.

Table 1. Values of H_{cJ} at various temperatures for VACODYM 3230. Calculation of β using Equations (2) and (3). The value of H_{cJ} at 20 °C is used as the low temperature value in both equations.

VAC 3230					
			$\beta = \frac{1}{H_{cJ}}\frac{\Delta H_{cJ}}{\Delta T}$	$\beta = \frac{\Delta\left(\ln H_{cJ}\right)}{\Delta T}$	
Temp (°C)	H_{cJ} (kOe)	ln (H_{cJ})	β (1/K)	β (1/K)	difference (%)
20	33	10.40			
60	25.2	10.13	-0.59%	-0.67%	14.1%
80	21.7	9.99	-0.57%	-0.70%	22.4%
100	18.5	9.83	-0.55%	-0.72%	31.7%
120	15.5	9.65	-0.53%	-0.76%	42.5%
150	11.5	9.35	-0.50%	-0.81%	61.8%
180	8.2	9.01	-0.47%	-0.87%	85.3%
210	5.4	8.59	-0.44%	-0.95%	116%

Table 2. Values of H_{cJ} at various temperatures for VACODYM 4514. Calculation of β using Equations (2) and (3). The value of H_{cJ} at 20 °C is used as the low temperature value in both equations.

VAC 4514					
			$\beta = \frac{1}{H_{cJ}} \frac{\Delta H_{cJ}}{\Delta T}$	$\beta = \frac{\Delta(\ln H_{cJ})}{\Delta T}$	
Temp (°C)	H_{cJ} (kOe)	ln (H_{cJ})	β (1/K)	β (1/K)	difference (%)
20	15	9.62			
60	10.27	9.24	-0.79%	-0.95%	20.1%
80	8.3	9.02	-0.74%	-0.99%	32.5%
100	6.6	8.79	-0.70%	-1.03%	46.6%
120	5.16	8.55	-0.66%	-1.07%	62.7%

The two tables above demonstrate the method people in industry use to find β, using equation (2). The large differences in the results produced by the two equations can clearly be seen, ranging from 14 to 116%. This difference increases as ΔT increases.

Besides using the parameter β to compare two competing materials, the parameter is also used to predict the value of H_{cJ} at an intermediate temperature, within the temperature range used to determine β. To predict the value of H_{cJ} at such a temperature, equations (2) and (3) must be rearranged. Rearranging equation (2) yields

$$H_{cJ}(T) = H_{cJ}(20°C)(1+\beta\Delta T) \tag{6}$$

And rearranging equation (3) yields

$$H_{cJ}(T) = H_{cJ}(20°C)e^{\beta\Delta T} \tag{7}$$

where $H_{cJ}(T)$ is the value of H_{cJ} at an intermediate temperature and $H_{cJ}(20\ °C)$ is the value of H_{cJ} at 20 °C. Equations (6) and (7) both assume that the lower temperature used for determining β is 20 °C. If a temperature range with a different lower temperature is used, then equations (6) and (7) must be adjusted accordingly.

Table 3. An example of how well each equation predicts the value of H_{cJ} at an intermediate temperature for VACODYM 3230. In this case a temperature range of 20 to 150 °C was used, with β=-0.50%/K for equation (6) and β=-0.81%/K for equation (7), as reported in Table 1.

VAC 3230			
Temp (°C)	H_{cJ} (kOe)	H_{cJ} (kOe) eq (6)	H_{cJ} (kOe) eq (7)
20	33	33.0	33.0
60	25.2	26.4	18.2
80	21.7	23.1	13.3
100	18.5	19.8	9.7
120	15.5	16.5	6.9
150	11.5	11.5	4.0

In the example above, equation (6) does a noticeably better job of predicting the value of H_{cJ} at intermediate temperatures. Using the data in Table 3, the sum of the squares of the difference between the experimental and calculated values was 5.83 for equation (6) and 327 for equation (7).

III. THE TEMPERATURE COEFFICIENT OF B_r

At this point, an obvious question to consider is: does the choice of equation to determine α affect the result, if we write a definition similar to equation (3)?

First, we need to write the analogous equation to equation (3) for the behavior of B_r, that is

$$\alpha = \frac{\Delta(\ln B_r)}{\Delta T} \tag{8}$$

The tables below show the calculation of α for the same materials used in Tables 1 and 2, comparing equations (1) and (8).

Table 4. Values of B_r at various temperatures for VACODYM 3230. Calculation of α using Equations (1) and (8). The value of B_r at 20 °C is used as the low temperature in both equations.

VAC 3230					
			$\alpha = \frac{1}{B_r} \frac{\Delta B_r}{\Delta T}$	$\alpha = \frac{\Delta(\ln B_r)}{\Delta T}$	
Temp (°C)	B_r (kG)	ln(B_r)	α (1/K)	α (1/K)	difference (%)
20	11.5	9.35			
60	11.08	9.31	-0.091%	-0.093%	1.9%
80	10.84	9.29	-0.096%	-0.099%	3.0%
100	10.59	9.27	-0.099%	-0.10%	4.2%
120	10.31	9.24	-0.10%	-0.11%	5.6%
150	9.87	9.20	-0.11%	-0.12%	7.8%
180	9.38	9.15	-0.12%	-0.13%	10.5%
210	8.85	9.09	-0.12%	-0.14%	13.7%

Table 5. Values of B_r at various temperatures for VACODYM 3230. Calculation of α using Equations (1) and (8). The value of B_r at 20 °C is used in both equations as the low temperature.

VAC 4514					
			$\alpha = \frac{1}{B_r} \frac{\Delta B_r}{\Delta T}$	$\alpha = \frac{\Delta(\ln B_r)}{\Delta T}$	
Temp (°C)	B_r (kG)	ln(B_r)	α (1/K)	α (1/K)	difference (%)
20	13.6	9.52			
60	13	9.47	-0.11%	-0.11%	2.27%
80	12.66	9.45	-0.12%	-0.12%	3.62%
100	12.3	9.42	-0.12%	-0.13%	5.11%
120	11.92	9.39	-0.12%	-0.13%	6.74%

In both cases above, the differences are much less significant than was observed in Tables 1 and 2. However, they do show the same trend as the β calculation, an increase in the difference as ΔT increases.

IV. DISCUSSION

Equations (1) and (2) are like the well-known equation used to describe the thermal expansion of materials.

$$\varepsilon = \frac{1}{L_0} \frac{\Delta L}{\Delta T} \tag{9}$$

In fact, this type of expansion is often called *linear* expansion, in part because it is one-dimensional, but also implying that the effect is mathematically linear with temperature. Magnetic properties are rarely linear as the temperature varies, so relying on equations that assume linear behavior is an invitation to potentially confusing results.

The following figures show the behavior of B_r and H_{cJ} with temperature, an equation fitted to each data set and a correlation factor, R^2.

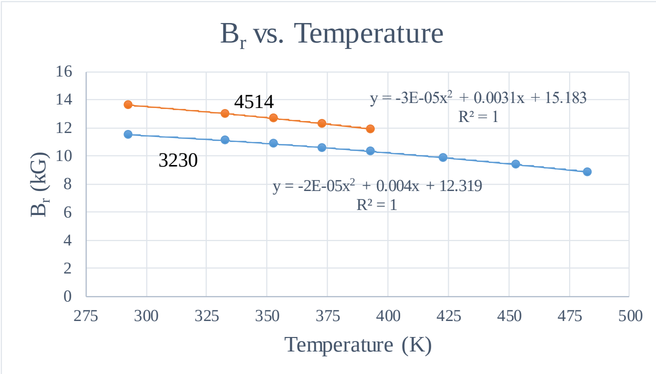

Fig. 1. Plots of B_r vs. temperature for VACODYM 3230 and 4514.

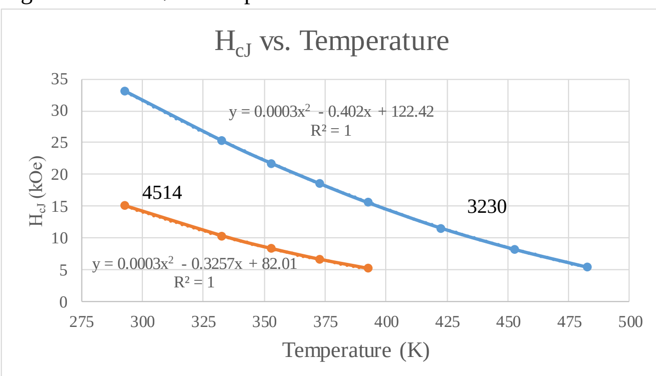

Fig. 2. Plots of H_{cJ} vs. temperature for VACODYM 3230 and 4514.

Fitting equations to each set of data in both graphs gave excellent results ($R^2 = 1$) with a second order polynomial. Trying to fit other types of functions, e.g. exponential, linear, logarithmic and power were all good, ($R^2 > 0.96$) but not as good as a simple second order polynomial.

The small square term in all the fitted equations highlights why equations (1) and (2) have some trouble fitting the data of real materials. Although the square term is less significant in Figure 1 for the B_r data, the influence of the square term is easier to see in Figure 2 for the H_{cJ} data, and demonstrates why equation (2) will give slightly different results, depending on the temperature range selected.

The poorer fit of the exponential and logarithmic functions was unexpected. It has been suggested that equation (3) is more physically correct than equation (2) and therefore ought to give better results. But that did not happen in the examples examined here. One assumption used in the derivation of both equations (3) and (7) is that β is completely independent of temperature. [Wikipedia 2017] That assumption is likely not valid in this case, and perhaps invalid for permanent magnets more generally. This assumption may be the cause of equation (3) not fitting the data as well as expected. A similar comment would also apply to α and equation (8).

The choice of units, SI or CGS, does not affect the determination of the temperature coefficient in any of the equations in this paper.

V. CONCLUSION

The paper shows that equation (3) is mathematically equivalent to equation (2), provided β is being measured at a single temperature, or over a small temperature differential. However, when these two equations are applied over a large temperature differential, as they routinely are, they yield divergent values of β. This is an undesirable situation. In our examples, equation (3) consistently yields a larger absolute numerical value of β than equation (2). In addition, equation (6) does a better job of predicting intermediate values of H_{cJ} than equation (7). Because nearly all data found in the literature for β are based on equation (2), it should be favored over equation (3) for reporting the temperature coefficient of H_{cJ} in permanent magnets in order to make rigorously valid comparisons and more accurate predictions of H_{cJ} at intermediate temperatures.

An analogous situation is also found for the reversible temperature coefficient of B_r; however, the discrepancy is much less significant.

At the very least, authors should reveal which equations are being used to determine the reversible temperatures coefficients, as well as the temperature range of the measurement, for the sake of transparency.

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https://en.wikipedia.org/wiki/Temperature_coefficient (accessed November 21, 2017) This reference also shows a mathematical connection between equations (6) and (7) via a Taylor series expansion of equation (7), for small values of ΔT.